Design Documents for an Open Online Developmental Math Course

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Course Justification

Introduction

Math is often used as a gatekeeper, "it has been used to stratify students, affording privilege to some and limiting opportunities for others (Martin et al., 2010, p. 14). These types of courses serve not merely as academic milestones but as critical junctures where varying backgrounds, preparedness levels, and educational goals converge. In environments such as developmental, remedial, or pre-college mathematics classrooms, the challenge intensifies as educators strive to meet the needs of students from diverse backgrounds, often marginalized in the realm of mathematics education. My aim is to reduce or even remove a barrier that persists for those who are already trying to have their relearning of math getting in the way of their already full lives.

I use the term developmental where remedial has been used in the past in a similar way. I prefer developmental and refrain from using remedial in either sense with the idea that it could imply there is something wrong with the student to be remedied. However, I do like the reclamation of the word usage by Bahr "Thus, remediation is, by definition, a ''remedy'' intended to restore opportunity to those who otherwise may be relegated to meager wages, poor working conditions, and other consequences of socioeconomic marginalization" (2008) because it gives power back the student. In this context, 'developmental courses' refer to foundational educational programs designed to equip students with essential skills and competencies, as mandated by an educational authority. These competencies are prerequisites for enrolling in gatekeeper courses that are advanced courses identified as pivotal requirements for ensuring that students can effectively participate in society and excel in their chosen fields by said authority. Thus, I want this course to be a means to remedy student's past barriers as a free and open resource to all.

Broader Reach:

This course, once fully developed, would be useful for all people who want to build their mathematical competencies to a level that will help them better function of their own volition within our society. This course can also be used as a refresher course for those who are going into a College Algebra course to better prepare themselves or it could be used as a practice to re-enforce points where they struggle. One of the major tenets of this project is that it is free and accessible to anyone.

Subject of Instruction:

Developmental mathematics for adults. What my course is intending to do is prepare those from no background to be ready and able to take the mathematics portion of the GED/HISET[1]. The GED/HISET is a test that can give high school equivalency but there are few supports for those who are studying independently. This also is the transition point to college mathematics and will serve those heading into or re-entering college mathematics.

Description of the Learners:

The learners would be adults who have been through compulsory education and are seeking to rebuild and re-engage with mathematics. This could be a pre-requisite course for college mathematics, and this could be a GED/HISET math preparatory class. Among all undergraduates who take any developmental courses, they take an average of 2.6 courses and has been estimated to be a total direct cost of 7 billion dollars to students (Scott-Clayton et al., 2014, p. 371). This trend is exacerbated for first-generation students, English as Second Language students, Black students, American Indian/Alaska native, and Latino students, who are disproportionately placed in developmental courses and less likely to complete them (Hodara, 2019, p. 1; Uretsky et al., 2021 p. 7). These courses serve as financial inhibitors,

academic barriers, and they have an opportunity cost when many institutions do not offer credit for completing these courses. I hope that a free and open course that is self paced (but designed for 16 weeks) will provide some alleviation from barrier to entry.

Learning Objectives:

Basic Math Operations

- Addition, Subtraction, Multiplication and division using a calculator
- Fractions, decimals, percentages, roots, and exponents

Geometry

- Use formulas related to shapes and objects (not memorizing)
- Be able to calculate surface area, radius, and diameter using given formulas of mathematical objects

Algebra

- Determine the value of a variable in an equation. Also understand how to write a formula with a variable based on a word problem

Graphs and Functions

- Read and analyze information in graphs and charts
- Understand common ways data is organized and use tables
- Understand concepts of median, mean, mode, range, probability, and statistics

Prerequisites required:

Ideally, mathematically none. However, there will be a need for a level of literacy needed, such as an 8th grade reading level.

Content resources:

- Website to host class (or learning management software such as Canvas)

- Video lessons

    o Including in video quizzing

- Practice Problem Sets

- Discussion Boards or Forum Component

- Video hosting for student work (possibly)

- Curriculum Guides

Instructional Ideas

The focus of this will be on rebuilding upon previous knowledge that the student already must engage with topics they have seen before. They will have some concept of some of the mathematical ideas present in this course. Where students who encounter ideas they 'met-before' influence relearning by either supporting or hindering it, depending on the context. (McGowen & Tall, 2010). What I hope that this class can do is take the students' ideas and have them test those preconceived notions, keep the ones that work for them already, and challenge the ones that don't so they can build more fruitful operation-ability. How this manifests itself are still a bit nebulous but here are two considerations.

In the structure there will be videos for the students to watch which will have pause points with questions for them to answer. At the end of the video the number correct will determine the next course of action available to the student. Whether that is continuing on in the curriculum or sent to another video where alternative methods are shown that they are either more familiar with or will find better illustrate their natural thinking.

Another idea is to have them teach their methods as well. How interactive this is with other students or myself is to be determined. At the base level it would involve them taking the time to create a script (or some other deliverable, such as a video) walking through a problem of their own creation with their own reasoning. This is called Problem Posing within mathematics education. And through problem posing students can and will respond in ways that reflect their personal commitments and values (Silver, 1994, p. 26). What this could mean is that that product can then be used by the next student in line as a reference or for critique, then they create their own for the next student, and so on. This would keep the course evergreen and center the student voice.

I also want to build this class in a manner that is self-sustaining with those who are interested can join at any time of their convenience outside of the university timeline. So those who are preparing for college or finished their high school equivalency can use this anytime. I also hope that this ends up being a free resource for anyone to join.

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Syllabus

Welcome to this self-paced class! It is intended for you to be able to build the skills of self-guided learning to inform your mathematical learning process.

You have to start from the beginning, no matter what the case. You cannot skip forward at any point. This course will show you different ways of using mathematics that you may have never seen before that you may find useful in your journey. I promise you there will be a point where you are humbled, I know I have. But keep going.

As you progress you will see that you will have re-enforcement of previous topics, moments of self-reflection, and other curricular activities that you may not be used to within a math course. I ask you to keep an open mind and willingness as you progress. You will see how far you have come by the end.

There are others who may be taking the course or have been taking the course. So, you will find artifacts of those who are in a similar situation as you. You will have opportunities to engage with those peers and get feedback as well. You will never be alone in this.

Who is this class for?

You are not expected to have any math pre-requisite going into this.

This is class is for people who:

1) Plan to take a high school equivalency test
   a) GED
   b) HiSet
2) Want to prepare for college math course
   a) Also, those who are taking entrance exams

If you fall into category 1), I suggest that you plan to take the full 4 months to do this course. It can be done quicker but this way you can take the course and plan to take the test when you are ready.

If you fall into category 2) I suggest you start this course in May before the fall semester or in August before the spring semester, depending on when you are taking the course.

Weekly Course Structure

Since this is a course where there is not a set meeting time, you are free to do work as you please in a manner that is convenient to you. You can start or end any time. The week will begin on Sundays and end on Saturdays. It is a 16-week course, which has due dates throughout to keep pace. 16 weeks or 4 months may seem like a long time, but this course is trying to span the entire curriculum. I hope you find it worth it.

Overall Course Structure

Unit 1	Math Operations	Assessments
Week 1	Addition and Subtraction	- Initial Discussion and reflection - Pre-test - Syllabus Quiz - Videos/Quiz - Problem Posing
Week 2	Multiplication and Division	- Videos/Quiz - Problem Posing
Week 3	Fractions and Decimals	- Videos/Quiz - Problem Posing
Week 4	Roots and Exponents	- Videos/Quiz - Problem Posing - Reflection Discussion - Unit Test
Unit 2	**Geometry**	**Assessments**
Week 5	Formulas related to shapes	- Videos/Quiz - Problem Posing
Week 6	Measurement (area, radius, diameter)	- Videos/Quiz - Problem Posing - Reflection Discussion Unit Test
Unit 3	**Algebra**	**Assessments**
Week 7	Variables	- Videos/Quiz - Problem Posing

Week 8	Solving Equations	- Videos/Quiz - Problem Posing
Week 9	Word Problems	- Videos/Quiz - Problem Posing - Reflection Discussion Unit Test
Unit 4	**Graphs and Functions**	**Assessments**
Week 10	Read and Analyze Graphs	- Videos/Quiz - Problem Posing
Week 11	Organized Data and Tables	- Videos/Quiz - Problem Posing
Week 12	Averages 1: Median and Mode	- Videos/Quiz - Problem Posing
Week 13	Averages 2: Mean and Range	- Videos/Quiz - Problem Posing
Week 14	Probability and Statistics	- Videos/Quiz - Problem Posing - Reflection Discussion - Unit Test
Unit 5	**Testing**	**Assessments**
Week 15	Practice	- Practice Tests by unit - Final Reflection - Problem Posing - Test Taking Strategies
Week 16	Practice	- Pre-Test Corrections - Final Test - Problem Posing

Assessments

Pre-Test

There will be a pre-test on all of the material for the course. The point of this is to show you how much you have gained in the process. By the end of the course, you will be asked to do corrections on the pre-test you took. This should show you how much you have overcome.

You do not need to attempt every problem.

Pre-Quiz

Each week you will have an initial quiz with a set of problems that are the expected goal of the END of the week. This way you have a way to see each week how you have grown. This is important for your growth mindset. Do not expect to do perfectly. If you do, then the hope is that by the end of the week you will have been able to find efficiencies in your process.

To move on, you would have to attempt each problem.

Video/Quiz

There will be videos each week to model some variations of the operation expected. There will be conceptual demonstrations to algorithmic. During these videos there will be questions for you to pause and answer and attempt again. With information embedded during the question process to help you along or link you to additional resources. The videos will also encode previous material, so you are continuously practicing what you have seen before.

To move on you have to get 100% but you only must retry questions you did not get.

Reflections

At different times during the course, you will have a reflection discussion question about your personal journey and what you are thinking. This is important! You are in charge of your own journey. You can only measure yourself against who you were yesterday. These reflections serve to see how far you have come and what you have overcome. You will also be able to see how others have overcome their obstacles in relearning math.

To move on you must submit a reflection and comment on two

Problem Posing

At the end of each section, you will be asked to create your own problems. Essentially problems that could replace the ones you had during that week. Creating your own exercises, or ones for others to do, demonstrates a high level of understanding of the material. I want you to be at that high level as you go on to the next topic or adventure. This will also give you an opportunity to solve the problems others have posed so that you have a large supply of practice problems.

To move on you must submit a problem, its solution, and answer 3 others

Practice Tests

In the last weeks you will have access to practice tests. In the first week, these tests will have problems where if you are unable to solve a problem, it will link you to a previous section that it is correlated to. In the second week you will have the opportunity to retake the tests until you get 100%, only on the problems you missed. In the final week you will also create your own full practice test with the problems you made earlier in the course and solve it yourself.

This is the end, there is no grade, once you get here you will have access to everything for reference and future practice as needed.

Tools Allowed

This is a self-propelled course. No one is watching you; you can use what you would like. ***But*** if you use outside resources that will not be available to you during the test on your own, then what is the point? The only tools allowed for the equivalancy tests are a calculator and formula sheet. However, if you are working towards a introductory college course, these are similar stipulations.

Canvas Site: Details

I am going to use the canvas site provided to build the structure for the course, but I also want to think about it without limitations first because there will be points where I am not knowledgeable enough about canvas that will cause hiccups. For example, can I have the course start at any time or can I have the deadlines be adapted to when the student starts?

Initial Conditions

- All of the course is mandatory from start to finish.
- I want them to only be able to access the next item within a module after they complete the previously assigned task.
- After a task is completed, they will have access to it going forward. It will be for reference, it will be for future tasks, and it will be there if they are stuck moving forward, they can take a step back.
- Try to have about 60 minutes of video each week of the course. The quizzes are going to be built into the videos so they will eat up most of the time of the curriculum. The topics listed will be combined into videos that seem cohesive. I would like to keep the videos under 12 minutes a piece to keep attention.
    - I will break up how the topics fit into videos once I begin production.

Definitions

- Tasks are all listed and described within the syllabus
    - Video/Quiz
        - Place holder videos will be used on topics from Khan Academy, then backfilled so they have a coherent structure and plan with similar pedagogical considerations taken for each one. Currently the title topics are associated with common core standards for which there is publicly available material for.
    - Reflection
    - Problem Posing
    - Pre-Quiz

- o Practice Tests
- o Pre-Test

Structure of Curricula

This was the most laborious process yet. Breaking down the topics beat by beat, all the way through. Creating videos will be a lot of work, but most can be filled in with YouTube videos for the time being.

Module – Unit 1: Math Operations

- Week 1 – Addition and Subtraction
  - o Addition Topics
    - Intro to addition
    - Addition strategies
    - Addition algorithms
    - Using place value to add 3-digit numbers (1 and 2)
    - Breaking apart 3-digit addition problems
    - Adding 3-digit numbers (regrouping)
    - Adding and subtracting on number line
  - o Subtraction Topics
    - Intro to subtraction
    - Subtraction strategies
    - Subtraction algorithms
    - Subtracting: Regrouping from zero
    - Subtracting: three-digit numbers and regrouping
    - Subtracting: regrouping twice
- Week 2 – Multiplication and Division
  - o Multiplication Topics
    - Equal Groups
    - Introduction to multiplication
    - Multiplication as Equal Groups
    - Multiplication as repeated addition
    - Multiplication on Number line
    - More ways to multiply

- Multiplication with Arrays
- Multiplication in real world contexts
- Commutative property of multiplication
- Distributive Property when Multiplying
- Properties and Patterns for Multiplication
  - Division Topics
    - Division as equal groups
    - Visualizing division with arrays
    - Division in context
    - Relating division to multiplication
    - Multiplication word problem and Division word problem
      - Parking lot, school building, blueberries, soda
    - Introduction to remainders
    - Division with area models
    - Introduction to division with partial quotients
      - No remainder and remainder
    - Intro to long division no remainder
    - Division word problem
      - Field goals and pizza
  - Multiplication and Division
    - Unknowns with multiplication and division
    - Associative property of multiplication
    - Using associative to simplify
- **Week 3 – Fractions and Decimals**
  - Fractions Introduction
    - Intro to fractions
    - Cutting shapes into equal parts
    - Identifying unit fractions word problems
    - Fractions in context
    - Identifying numerator and denominator
    - Recognizing fractions
    - Recognizing fractions greater than one
    - Relating number lines to fraction bars
    - Fractions on a number line
    - Whole numbers as fractions
    - Writing whole numbers as fractions
    - Comparing fractions visually
    - Comparing fractions with the same numerator

- Creating equivalent fractions
- Arithmetic with Fractions
  - Finding common denominators
    - ¼, 5/6, 3/5, 7/2
  - Comparing fractions 1
    - Unlike denominators
  - Comparing fractions word problems
  - Decomposing a fraction
  - Decomposing a mixed number
  - Adding fractions with like denominators
  - Subtracting fractions with like denominators
  - Fraction word problems
  - Multiplying fractions and whole numbers visually
  - Fraction multiplication on the number line
  - Visually adding fractions (5/6+¼ and so on)
  - Adding fractions with unlike denominators introduction
  - Adding fractions with unlike denominators
  - Subtracting fractions with unlike denominators introduction\
  - Adding fractions word problems – paint
  - Subtracting fractions word problem: tomatoes
  - Understanding fractions as division
  - Creating a fraction through division
  - Multiplication and division relationship for fractions
  - Dividing a unit fraction by a whole number
  - Dividing a whole number by a unit fraction
  - Fraction division in context
  - Understanding division of fractions
  - Writing fraction division word problems
  - Meaning of the reciprocal
- Decimals
  - Writing a number as a fraction and a decimal
  - Writing decimals and fractions greater than 1 shown on grids
  - Writing decimals and fractions shown on number lines
  - Rewriting fractions as decimals
  - Decimals as words
  - Relating Decimals and fractions in words
  - Decimal place value
  - Plotting decimal numbers on a number line

- Decimal place value with regrouping
- Rewriting decimals as fractions
- Common fractions and decimals
- Week 4 – Roots and Exponents
  - Exponents
    - Intro to exponents
    - The zeroth power
    - Exponents of decimals
    - Powers of fractions
  - Order of Operations
    - Order of operations introduction
    - Worked example: Order of operations (PEMDAS)
    - Order of operations examples: exponents
    - Comparing exponent expressions
    - Order of operations example: fractions and exponents
  - Roots
    - Intro to square roots
    - Intro to cube roots
    - Square root of a decimal
    - Exponent properties
      - With products
      - With parentheses
      - With quotients

Module – Unit 2: Geometry

- Week 5 - Formulas related to shapes
  - Intro to Area
    - Intro to Quadrilaterals
    - Intro to area and unit squares
    - Measuring rectangles with different unit squares
    - Creating rectangles with a given area 1 and 2
    - Counting unit squares to find area formula
    - Transitioning from unit squares to area formula
    - Area with partial grids
    - Finding missing side when given area
    - Comparing areas of plots of land
    - Area and the distributive property
  - Perimeter

- Perimeter Introduction
- Perimeter of a shape
- Finding perimeter when a side length is missing
- Finding missing side length when given perimeter
- Perimeter word problems
  - Table and Skating rink
- Comparing areas and perimeters of rectangles
  - Odds and Ends
    - Telling time with number line
    - Time differences example
    - Time word problem: travel time
    - Time word problem: puzzle
  - Area and Perimeter
    - Area & perimeter word problems
  - Pythagorean Theorem
    - Intro to Pythagorean theorem
    - Pythagorean theorem example
    - Pythagorean Theorem word problem: carpet and fishing boat
- Week 6 - Measurement (area, radius, diameter, volume)
  - Mass
    - Understanding Mass
    - Word problems with mass
  - Triangles
    - Area of a triangle
    - Finding area of triangles
    - Triangle missing side example
  - Area of Composite Figures
    - Finding area by rearranging parts
    - Area of composite shapes
  - Volume
    - Volume intro
    - Understanding volume
    - Measuring volume as area times length
    - Volume of a rectangular prism
    - Word problems with volume
    - Volume through decomposition
    - Volume word problem: water tank
  - Surface Area

- Intro to nets of polyhedra
- Surface area using a net: rectangular and triangular prisms
- Surface area word problem example
    - Circles
        - Radius, diameter, circumference & pi
        - Labeling parts of a circle
        - Radius and diameter from circumference
        - Relating circumference and area
        - Area of a circle
        - Finding circumference of a circle when given the area
        - Area of shaded region
        - Volume word problem: gold ring
        - Volume of triangular prism and cube
    - Volume continued
        - Cylinder volume and surface area
        - Volume of sphere
        - Volume of Cone

Module – Unit 3: Algebra

- Week 7 – Variables
    - Algebraic Thinking
        - Constructing numerical expressions
        - Evaluating expressions with and without parentheses
        - Translating expressions with parentheses
        - Intro to rates
        - Solving unit rate problem
        - Rate problems
        - Comparing rates
        - The meaning of percent
        - Percents from fraction models
        - Fraction, decimal and percent from visual model
        - Finding a percent
        - Percent word problem: recycling cans
    - Negative numbers
        - Introduction to negative numbers
        - Negative symbol as opposite
        - Number opposite
        - Negative decimals on the number line

- - - Ordering negative numbers
  - o Variables
    - What is a variable?
    - Terms, factors, and coefficients
    - Why aren't we using the multiplication sign?
- Week 8 – Solving Equations
  - o Substitution of values
    - Evaluating an expression with one variable
    - Evaluating exponent expressions with variables
    - Evaluating expressions like $5x^2$ and $1/3(6)^x$
    - Evaluating expressions with two variables
    - Expression value intuition
    - Evaluating expressions with variables (exponents)
  - o Algebraic Expressions
    - Writing basic expressions with variables
    - Writing algebraic subtraction expressions
    - Writing expressions with variables
    - Writing expressions with parentheses
    - Writing expressions with variables and parentheses
    - Writing basic expression word problems
    - Same thing to both sides of equations
    - Representing a relationship with an equation
    - Dividing both sides of an equation
    - One-step equations intuition
  - o Two Step Equations and Beyond
    - Same thing to both sides of equations
    - Intro to two-step equations
    - Two-step equations intuition
    - Worked example: two-step equations
    - Two-step equations with decimals and fractions
    - Find the mistake: two-step equations
    - Intro to equations with variables on both sides
    - Equations with variables on both sides
      - Two examples
    - Equation with variable in denominator
- Week 9 – Word Problems
  - o Worked Examples:
    - Equation word problem: super yoga

- Equation word problem: computers
- Equation word problem: garden
- Equation word problem: oranges

Module – Unit 4: Graphs and Functions

- Week 10 – Read and Analyze Graphs
  - Coordinate Plane
    - Points on a coordinate plane example
    - Plotting a point (ordered pair)
    - Finding the point not graphed
  - Solutions Through Graphing
    - Solutions to 2-variable equations
    - Worked example: solutions to 2-variable equations
    - Completing solutions to 2-variable equations
    - Intro to intercepts
    - x-intercept of line
    - Intercepts from an equation
    - Worked example: intercepts from an equation
    - Intro to slope
    - Slope and direction of a line
    - Positive and negative slope
    - Worked example: slope from two points
    - Slope from equation
- Week 11 – Organized Data and Tables
  - Graphs
    - Creating picture and bar graphs
    - Solving problems with picture graphs
    - Interpreting picture graphs
      - Paint and notebook
    - Reading bar graphs – movie
    - Interpreting bar graphs – color
  - Line plots
    - Measuring lengths to nearest unit
    - Graphing data on line plots
    - Interpreting line plots with fractions
  - Organizing data
    - Statistical Questions
    - Representing data

- Frequency tables and dot plots
- Creating a histogram
- Interpreting a histogram

- Week 12 – Averages 1 Median, Mean, mode and Range
  - Statistics intro: Mean, median and mode
  - Mean median and mode example
  - Missing values given the mean
  - Impact on median and mean: removing an outlier
  - Impact on median and mean: increasing an outlier
  - Median and range puzzlers
  - Interquartile range
  - Reading box plots
  - Constructing a box plot
  - Worked example: creating a box plot
  - Worked example: creating a box plot
  - Interpreting box plots

- Week 13 – Averages Continued
  - Mean absolute deviation
  - Mean absolute deviation example
  - Comparing dot plots, histograms, and box plots
  - Word problems

- Week 14 – Probability and Statistics
  - Shapes of distributions
  - Clusters, gaps, peaks and outliers
  - Intro to theoretical probability
  - Simply probability: yellow marble, non-blue marble
  - Experimental probability
  - Intuitive sense of probabilities
  - Making predictions with probability
  - Probability models example: frozen yogurt

Module – Unit 5: Testing

- Week 15 – Testing
- Week 16 – Testing